An *instance* of a quantified sentence is a sentence where the quantifier has been removed and all the instances of the variables bound by the quantifier have been replaced by a constant.

E.g., $\mathsf{Fc} \wedge \mathsf{Gc}$ is an instance of $\forall \mathsf{x}(\mathsf{Fx} \wedge \mathsf{Gx})$. $\neg \mathsf{Hd}$ is an instance of $\exists \mathsf{x} \neg \mathsf{Hx}$.

$\forall$E (Universal Quantifier Elimination)

Given:

$\Gamma \quad \vdash \forall \upsilon \varphi(\upsilon)$

can infer:

$\Gamma \quad \vdash \varphi(\kappa)$,

for any constant κ.

> Look for a sequent with a universally quantified succedent; infer a sequent whose succedent is any instance of the universally quantified sentence. Keep datum unchanged.

$\vee$I (Existential Quantifier Introduction)

Given:

$\Gamma \quad \vdash \varphi(\kappa)$ (κ a constant)

can infer:

$\Gamma \quad \vdash \exists \upsilon \varphi(\upsilon)$.

> Given a sequent whose succedent involves a constant, infer a sequent whose succedent is an existentially quantified sentence of which the original succedent is an instance. Keep the datum unchanged.

∧I (Universal Quantifier Introduction)

Given:

$\Gamma \qquad \vdash \varphi(\kappa)$

can infer:

$\Gamma \qquad \vdash \forall \upsilon \varphi(\upsilon)$,

provided κ does not appear in any of the sentences in Γ.

> Find a sequent whose succedent involves a constant; infer a sequent whose succedent is a universally quantified sentence of which the original succedent is an instance. Keep datum unchanged. Proviso: the constant in question must not appear in the datum.

∨E (Existential Quantifier Elimination)

Given:

$\Gamma \qquad \vdash \exists \upsilon \varphi(\upsilon)$

$\Delta, \varphi(\kappa) \qquad \vdash \psi$

can infer:

$\Gamma, \Delta \qquad \vdash \psi,$

provided κ does not appear in any of Γ, Δ, and ψ.

> Find a sequent with an existentially quantified succedent, a sequent with an instance of that existentially quantified sentence in the datum; infer a sequent whose succedent is the succedent of the second sequent, and whose datum is the combination of the two initial datums with the instance of the existentially quantified sentence removed. Proviso: the constant of the instance must not appear anywhere in the inferred sequent.